Dynamycal Systems in modern geometry, topology and physics (9 ECTS)

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2e semestre

Présentation

Dynamical systems in our course will be presented mainly not as an independent branch of mathematics but as a very powerful tool that can be applied in geometry, topology, probability, analysis, number theory and physics. We consciously decided to sacrifice some classical chapters of ergodic theory and to introduce the most important dynamical notions and ideas in the geometric and topological context already intuitively familiar to our audience. As a compensation, we will show applications of dynamics to important problems in other mathematical disciplines. We hope to arrive at the end of the course to the most recent advances in dynamics and geometry and to present (at least informally) some of results of A. Avila, A. Eskin, M. Kontsevich, M. Mirzakhani, G. Margulis.
In accordance with this strategy, the course comprises several blocks closely related to each other. The first three of them (including very short introduction) are mandatory (with exception for topics highlighted by blue color). The decision, which of the topics marked with asterisks and highlighted by blue color would be selected, would depend on the background and interests of the audience.

Programme

2.1. **Introduction.** We will introduce dynamical systems using the most elementary examples — rotation of the circle and continued fractions.

2.2. **Dynamics and geometry.** In this part we will illustrate how dynamical methods can be used to study one of the most classical notions of differential geometry—geodesics on surfaces of negative curvature. Here is our approximate plan :

1) Introduction to hyperbolic geometry. M.bius transformations. Fuchsian groups.
2) Geodesics on surfaces of negative curvature. The geodesic flow and its properties.
3) Geodesic flow on modular curve as a continued fraction map.
4) Teichmuüller space. Teichmuüller geodesic flow.
5) Counting of simple closed geodesics: results by M. Mirzhakhani.

2.3. **Dynamics and topology.** In this part we stay again on a Riemann surface but now we would like to have an almost flat metric on it and to consider related geodesic flow (equivalently, we study measured foliations on such a surface). The purpose of this block is to make a crash course in ergodic theory with a topological interpretation of the main notions and results. The plan is as follows :

1) Interval exchange transformations (IET) as natural generalizations of continued fractions.
2) IET as the first return maps on transversal for measured foliations on oriented surface. Poincar. recurrence theorem.
3) Key ergodic properties: minimality, ergodicity, number of invariant measures (illustrated by IET).
4) Multiplicative ergodic theorem. Topological interpretation of Lyapunov exponents. Sums of Lyapunov exponents as uniform bounds for degrees of holomorphic subbundles.
5) Anosov and Pseudoanosov diffeomorphisms of surfaces. Introduction to hyperbolic dynamics (Markov partitions, invariant measures etc).
6) * IET and billiards in rational polygons. Counting billiard orbits of special types.
7) * Action of GL(2;R) on the moduli space of Abelian differentials. Ehrenfest wind tree model of Boltzmann gas.

2.4. * **Dynamics and number theory.** This block (that can be chosen by our auditory) is dedicated to homogeneous dynamics and its applications to famous conjectures in number theory, such as Oppenheim conjecture (solved) and Littlewood conjecture (still open). We mainly will follow G. Margulis work in this direction.

Connaissances requises

We expect our audience to be familiar with basic differential geometry, basic topology and basic measure theory.

Bibliographie

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