Telepathy

Two persons, a magician and his assistant, and a deck of 52 cards are requested to perform this trick. First of all, the magician leaves the room while the assistant asks five persons to take one card each. Then, he shows to the audience the card the magician will have to guess. He hides it, and he puts the four other cards in plain sight. The magician returns, looks at the four cards and guesses the hidden one.

For instance, if the visible cards are

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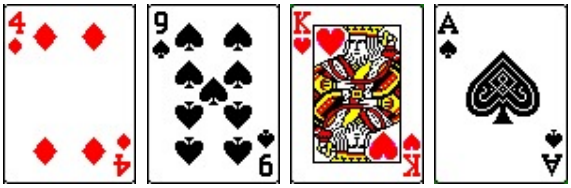
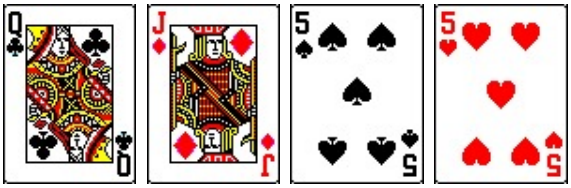


the magician answers ..... (8♢).

If the assistant shows



the hidden card is the ..... (5♣).

Of course, there is a trick. Can you explain it?

♣: clubs ♢: diamonds ♡: hearts ♠: spades

What do you suggest at first sight?

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Why is this always possible?

.....

.....

What looks important in the way to put the cards?

.....

How many ways have you got to order the three other cards? Can you prove it?

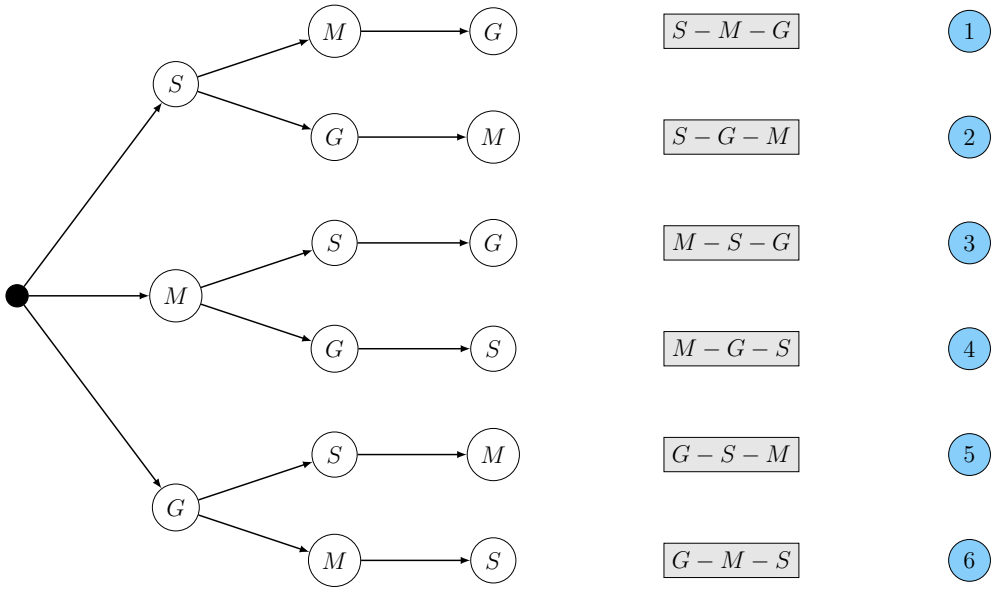
What looks important in the way to put the cards?

The magician looks at the order of the cards.
We have chosen the following order: $1 < 2 < 3... < 10 < J < Q < K$. And, when two cards have the same value, we order them thanks to their symbol: $\clubsuit < \diamondsuit < \heartsuit < \spadesuit$. For instance

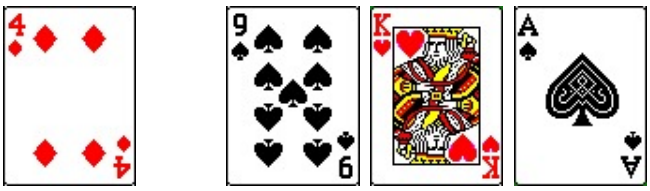
$$1\spadesuit \dots K\heartsuit \text{ and } 5\diamondsuit \dots 5\spadesuit.$$

How many ways have you got to order the three other cards? Can you prove it?

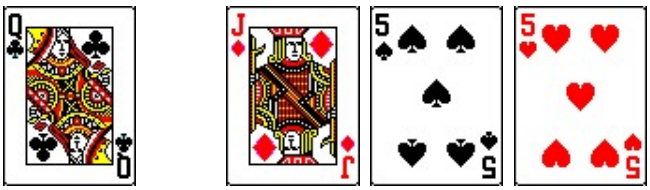
These three cards are ordered: Small $<$ Medium $<$ Great. There are six ways to order them.
Let us draw a tree to prove it:



In which order are these three cards in the examples at the beginning?



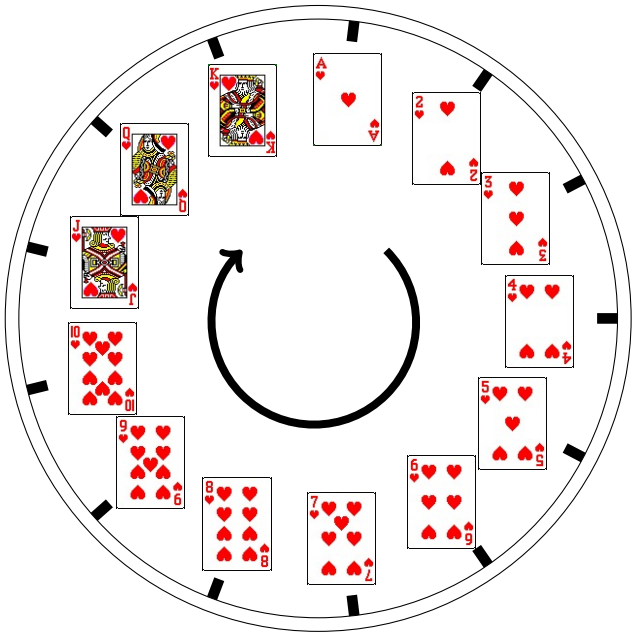
. . . . . .



. . . . . .

Now, can you imagine how to guess the hidden card from the first card on the left?

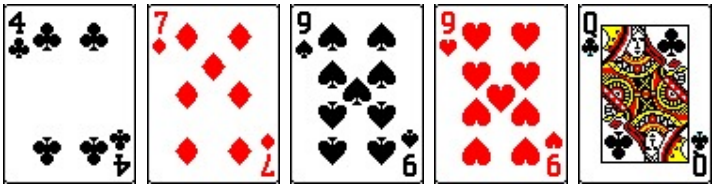
Use the following clock to compute the "distance" between the first card on the left and the hidden card in the examples at the beginning.



Why is this always possible?

. . . . . .

Now, it is up to you! Which card the assistant must hide if the public chooses the following ones:



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How will he order the four other cards?

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References: Les maths fantastiques (IREM de Paris).